NOTATION

d, spiral diameter; l, sag; b, turns per unit length of spiral; ΔT, temperature head between heater and surrounding air; U_0, amplitude of mechanical oscillations; θ_0, temperature oscillation amplitude; φ, phase shift between thermal and mechanical oscillations.

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NUMERICAL ANALYSIS OF TRANSFER PROCESSES IN SEMICONDUCTING DEVICES AND STRUCTURES.

1. GENERAL PRINCIPLES OF CONSTRUCTING SOLUTIONS OF THE FUNDAMENTAL SYSTEM OF EQUATIONS

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UDC 621.382.82.001:519.95

The basic situation is considered of constructing effective methods and algorithms of numerical analysis of transfer processes of charge carriers in semiconducting devices and structures.

Most transfer processes of energy, mass, momentum, charge, etc. can be described within contemporary science and technology only by systems of nonlinear partial differential equations. Computational methods taking into account the specifics of given physical problems and being the subject of a new scientific discipline, computational physics [1], have been developed and widely used so as to model these processes, being of significant scientific and practical interest.

The study of charge transfer processes in semiconducting devices and structures is one of the most important problems. Its complexity consists of the fact that transfer of charged particles under the action of an external electric field occurs in the presence of immobile charges and the internal electric field due to these charges.

The need to solve this problem is related to the present transition to submicron technology of integrated circuits, and consequently, to the difficulties in performing a real physical experiment studying the internal processes in the semiconducting structures, being the basis of these circuits.

In the present paper we consider both the difficulties generated in numerical analysis of integrated semiconducting structures and the ways of overcoming them. It is shown that integral finite-difference formulations of the Sharfetter–Gummel [2] and Engl–Dirks [3] type for the continuity equation are special cases of a general integral formulation, obtained on the basis of the G. I. Marchuk integral identity under a number of physical assumptions. By using this identity one can also obtain integral formulations for the case of including Fermi statistics. Two-stage methods of vector relaxation systems (VRS) are developed in concluding the introduction of the physical balancing principle of the iteration solution of the problem.

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 44, No. 2, pp. 284-293, February, 1983. Original article submitted November 23, 1981.

0022-0841/83/4402-0199$07.50

The problem under consideration of analyzing charge transfer processes in semiconducting devices and structures consists in the stationary case of solving the fundamental system of equations of the theory of semiconducting devices [5]:

$$\varepsilon\nabla^2\Psi = q(n - p - N_d + N_a); \tag{1}$$

$$\nabla\vec{j}_p = -qR; \tag{2}$$

$$\nabla\vec{j}_n = qR; \tag{3}$$

$$\vec{j}_p = -q\mu_p p\nabla\Phi_p; \tag{4}$$

$$\vec{j}_n = -q\mu_n n\nabla\Phi_n, \tag{5}$$

where the relation between the concentrations of mobile carriers n, p, the Fermi quasilevels Φ_n, Φ_p, and the electrostatic potential Ψ is established by means of some statistics. In the case of using Fermi statistics, for example, this is established by means of the Fermi integral [6]. Empirical equations [7] are usually used for the dependences of the mobilities μ_p, μ_n on the various factors (the field $\nabla\Psi$, the total concentration $N_d + N_a$, etc.). As recombination-generation models one usually chooses the Shockley–Read–Hall model [8]

$$R_{\text{Sh-R-H}} = (pn - p_0n_0)/[\tau_n(p + n_i) + \tau_p(n + n_i)] \tag{6}$$

and the Auger recombination model, for example, [9]

$$R_A = G(n^2p + p^2n), \tag{7}$$

where $G = 2\cdot10^{-37}$ m^6/sec.

In this case the general recombination-generation model will be

$$R = R_A + R_{\text{Sh-R-H}}. \tag{8}$$

The system under consideration (1)-(5) is nonlinear and is characterized by the large range of variation of the basic variables n and p (≈20 orders), which largely determines the difficulties occurring during numerical analysis of semiconducting devices and structures. To these belong:

1) the requirement of a large memory, being a consequence of the required number of points of spatial discretization, so as to guarantee sufficient accuracy and stability of approximating the original problem by a system of algebraic equations;

2) slow convergence of the traditional methods of Gummel-type solutions [10, 11] with the classical methods of selecting the initial approximation [11], where external voltages are given by powers in 1-2.5 kT/q;

3) the practical ineffectiveness of a powerful and universal numerical method, such as Newton's method, for solving an approximate system. This is related to the absence of both special iteration methods of solving linear systems of equations numerically in the multidimensional case, which would guarantee solutions with the required amount of reliability, and effective methods of selecting the initial approximation. Therefore, in solving linear systems one must use direct methods, which requires huge amounts of memory, consisting of dozens of megabytes, and large time expenditure.

Concerning the first point, we note that significant success was achieved by using the Sharfetter–Gummel formulation [2] for the continuity equations (2), (3). As was shown in [12] **for a model** problem (R = 0, μ = const), this approach, in conjunction with the usual finite-difference version of the Poisson equation, has high accuracy $O(h^2)$ in the output characteristics of the device, as was verified by the numerical experiments described in [13] (for $R \neq 0$).

We show now (for simplicity we use the one-dimensional case) that the Sharfetter–Gummel formulation is a special case of the formalism obtained on the basis of **the Marchuk integral** identity [4, 14] in conjunction with a number of physical assumptions. For this we introduce the following notation in Marchuk's approximate problem [14] for the electron continuity equations (3), (5) in **normalized** form ($q = n_i = 0$) and in the case of using Boltzmann statistics: 1) $p^M = -\mu_n \exp(\Psi)$; 2) $q^M = 0$; 3) $\varphi^M = \exp(-\Phi_n)$; 4) $f^M = R$; 5) $\Delta x_k = h_k$. This gives

$$-\frac{1}{h_k}\left\{\frac{[\exp(-\Phi_n)]_{k+1}-[\exp(-\Phi_n)]_k}{\int\limits_{x_k}^{x_{k+1}}[-\mu_n\exp(\Psi)]^{-1}dx}+\frac{[\exp(-\Phi_n)]_k-[\exp(-\Phi_n)]_{k-1}}{\int\limits_{x_{k-1}}^{x_k}[-\mu_n\exp(\Psi)]^{-1}dx}\right\}=\frac{1}{h_k}\int\limits_{x_{k-\frac{1}{2}}}^{x_{k+\frac{1}{2}}}R\,dx. \tag{9}$$

To obtain the Sharfetter–Gummel integral formulation (in the form of [15, 16]) of the continuity equation from (9) it is necessary to make the following physical assumptions (we recall that one was already made, the use of the Boltzmann statistics):

1) a linear variation of R in the segment $(x_{k+1/2}, x_{k-1/2})$, which makes it possible to approximate the recombination integral in (9) as follows on the uniform grid:

$$\int\limits_{x_{k-\frac{1}{2}}}^{x_{k+\frac{1}{2}}}R\,dx=h_kR_k;$$

2) the mobility changes weakly in the integration segments (x_{k+1}, x_k), (x_k, x_{k-1}), which makes it possible to take it out of the integral by using mean values near the points $x_{k+1/2}$, $x_{k-1/2}$;

3) linear variation of the electrostatic potential Ψ on the mesh [17]. As a result the integral

$$J=\int\limits_{x_k}^{x_{k+1}}[\exp(\Psi)]^{-1}dx$$

can be approximated on the uniform grid by the following expressions: a) when $\Psi_k \approx \Psi_{k+1}$, then [17, 18]

$$J=h_{k+1}[\exp(-\Psi_{k+1})+\exp(-\Psi_k)]/2;$$

b) when $\Psi_k \neq \Psi_{k+1}$, then [17]

$$J\approx -h_{k+1}[\exp(-\Psi_{k+1})-\exp(-\Psi_k)]/(\Psi_{k+1}-\Psi_k).$$

As noted in [14], the integrals appearing in Eq. (9) are not carried out in the general case. It is useful to use physical assumptions to approximate them. It was shown by Engl and Dirks [3] that in the one-dimensional case the recombination integral R can be selected by using more natural physical assumptions. As a result this approximation improves the accuracy in the current [3], which agrees with the conclusion [14] on the change in order of approximation of the problem from $O(h^2)$ to $O(h)$ when the smoothness of one of the functions $p^M(-\mu_n \exp \Psi)$, $q^M(0)$, $f^M(R)$ is violated (for the given R). We also note that the Marchuk integral relation (9) is accurate within $O(h^2)$ in the case of using quasiuniform steps in space [14], which simplifies considerably the choice of a grid of spatial discretization, though only for the continuity.

It is easily noted that the approximate Marchuk problem can also be used to construct approximate continuity equations when Fermi statistics has to be used. For this we use the continuity equation in the original form (3), (5), and use the following notation in Eq. (1.12) of [14]: 1) $p^M = \mu_n n$; 2) $\varphi^M = \Phi_n$; 3) $q^M = 0$; 4) $f^M = R$; 5) $\Delta x_k = h_k$.

As a result we obtain the general integral formulation of the continuity equation in the form

$$-\frac{1}{h_k}\left[\frac{(\Phi_n)_{k+1}-(\Phi_n)_k}{\int\limits_{x_k}^{x_{k+1}}(\mu_n n)^{-1}dx}-\frac{(\Phi_n)_k-(\Phi_n)_{k-1}}{\int\limits_{x_{k-1}}^{x_k}(\mu_n n)^{-1}dx}\right]=\frac{1}{h_k}\int\limits_{x_{k-\frac{1}{2}}}^{x_{k+\frac{1}{2}}}R\,dx, \tag{10}$$

where the relation of n with the electrostatic potential Ψ and with the Fermi quasilevel Φ_n is expressed by means of the Fermi integral [6]. Similar expressions can also be obtained for the continuity equation of holes in the form (2), (4).

Thus, to enhance the accuracy and stability of the approximate continuity equation it is necessary to use the integral formulation of equations of type (9), (10), special cases of which are (in the case of using Boltzmann statistics and a number of physical assumptions) the formulations of the Sharfetter–Gummel [2] or Engl–Dirks [3] type.

After approximating the original problem by systems of nonlinear algebraic equations, one encounters the problem of integral methods of their solutions. Two approaches exist for enhancing the effectiveness of solution algorithms of the given approximate problem by the use of integral methods: a choice of an initial approximation by using physical assumptions, and the development of new methods of solving the discrete analog of the system of equations.

It was shown in [19] that in the case of an initial approximation, obtained by numerical solution of the truncated system, the convergence of the Gummel method [10], possessing as a whole linear convergence [20], can behave as quadratic or nearly quadratic. In this case the assumption of quasiequilibrium [19] was used to obtain the truncated fundamental system. The conclusion on significant acceleration of the solution to the problem was also obtained thanks to using the STEPSOLVER method [3], in which a step-by-step construction of the simplified physical models was provided according to the device regions, thus approaching "exact" solutions. The drawback of the latter method is the need to provide initial information on the regions, where one model or another may be valid.

However, with the increasingly complicated working conditions of devices the convergence of solutions by the Gummel method [3, 19] slows down, therefore the development of new effective methods of solving the discrete analog of the fundamental system is quite important.

Due to the complications noted above, most promising for the analysis of integrated circuits are methods of vector relaxation systems [21], consisting of a partial system approach to solving partial differential equations, since in this case the systems of linear equations have a regular shape, and the application of traditional methods of the type of successive upper relaxation becomes justified [16]. The essence of the system approach to the solution consists of the following:

1) choice of a basis of fundamental and auxiliary variables, in whose terms the system of differential equations with auxiliary algebraic relations is described;

2) approximation of the equations, boundary conditions, and algebraic relations by the method of finite elements [14] or finite differences [22];

3) introduction of assumptions, if necessary, concerning some variables, with the use of additional relations so as to obtain the final form of solvable algebraic equations;

4) choice of an initial approximation for the approximate problem, having, as a rule, a multistage nature;

5) solution of the systems of algebraic equations, obtained as a result of satisfying steps 1-3, by the vector relaxation method, where the vectors are the values of all the unknowns at the grid sites of spatial discretization, with the use of iteration methods (multistage in the general case [23]) for finding these vectors.

The results of the calculations indicate the importance of choosing a basis of variables in which algebraic systems are formulated (steps 1-3) and the vector relaxation in step 5 is realized. A comparison was performed [21] of the Seidman–Choo method in the basis $\delta\Psi$, n, p, W_n, W_p, W with the method suggested in the basis $F = \exp(\Psi)$, $Q_n = \exp(-\Phi_n)$, $Q_p = \exp(\Phi_p)$, WQ_n, WQ_p, W, and the high effectiveness of the latter was shown. We note that to some extent the success achieved was related to eliminating the requirement of a good initial approximation in Ψ, automatically appearing in Gummel-type methods [10, 11], since the algebraic equations obtained from the Poisson equation (1) at the third stage of the system approach are replaced by linearized equations, augmented by the Boltzmann statistics. In this case, besides accelerating the convergence and widening the range of convergence, the requirement on a choice of initial approximation can be weakened.

For example, for the method of [21] it is sufficient to choose the initial approximation for the Fermi quasilevel, according to [19], and the electrostatic potential under the assumption of quasineutrality, interchanging in this case the order of points 4 and 2, 3 of the method.

Further acceleration of convergence can be achieved by using two-stage methods.

We divide the system (1)-(5) with the recombination law (6) into two subsystems (I and II). In the case of Boltzmann statistics, they have the following normalized shapes in the basis of fundamental variables $F = \exp \Psi$, $Q_p = \exp \Phi_p$, $Q_n = \exp(-\Phi_n)$ [21]:

subsystem I

$$W = \tau_n (Q_p F^{-1} + 1) + \tau_p (Q_n F + 1),\ W_{Q_p} = Q_p / W,\ W_{Q_n} = Q_n / W, \tag{11}$$

$$\nabla (F^{-1} \mu_p \nabla Q_p) - W_{Q_n} Q_p = -W^{-1}, \tag{12}$$

$$\nabla (F \mu_n \nabla Q_n) - W_{Q_p} Q_n = -W^{-1}, \tag{13}$$

and subsystem II

$$\nabla^2 (\ln F) = F Q_n - F^{-1} Q_p - N_d + N_a. \tag{14}$$

Consequently, in one of the subsystems there are two modified differential equations (12), (13) and three algebraic equations, solved for W, W_{Q_p}, W_{Q_n} (11); in the second there is only one differential equation, the Poisson equation (14).

Following a finite-difference approximation, the subsystems I and II consist of systems of nonlinear algebraic equations in the vectors F, Q_n, Q_p, W_{Q_n}, W_{Q_p}, W. We apply the vector relaxation method to the subsystems of algebraic equations I, II and to the equations of subsystem I. This will also be a two-stage VRS method [24], consisting of the following:

1) an initial approximation is assigned for Ψ, Φ_n, Φ_p corresponding to the device structure;

2) subsystem II is solved by Newton's method (at the first iteration of the two-stage VRS method, up to the required convergence, and at the subsequent iterations — a given number of times, which can be controlled [21]) for F for fixed Q_n, Q_p;

3) subsystem I is solved by the VRS method for fixed F: a) the values of W_{Q_p}, W_{Q_n}, W are calculated at fixed Q_n, Q_p; b) the system (12), (13) is solved for Q_n, Q_p for fixed W, W_{Q_n}, W_{Q_p}; c) items a) and b) of 3) are repeated a given number of times or till the required convergence;

4) items 2) and 3) of the two-stage VRS method are continued up to the required accuracy, determined either by $|\delta\Psi|_{\max_l}$, where l is the number of Newton iterations of item 2), or by the current criterion [19, 21]. We note that the choice of the criterion must depend on the purpose of the study.

It follows from the iteration scheme of the method that after satisfaction of item 3) we have agreement of the Q_n and Q_p values with fixed F (first stage), and, besides, there is consistency between Q_n and Q_p themselves (second stage). Equations (12), (13) in the variables Q_n, Q_p are related in item 3) only by expression (11), therefore this relation is determined by the recombination model and ingredients of "numerical recombinations" [21].

It should be noted that errors unavoidably generated in solving the linear systems of equations in item 3) can also be associated with "numerical recombination," as well as corresponding errors in item 2) — with "unbalanced charge" [21].

Thus, one of the VRS processes is used in the given case to eliminate the dependences of W, W_{Q_n}, W_{Q_p} on Q_n, Q_p, and, consequently, for transforming from the basis F, Q_n, Q_p, W, W_{Q_n}, W_{Q_p} to the basis F, Q, where $Q = (Q_n, Q_p)$.

For practical realization of the method it is convenient to transfer from the basis Q_n, Q_p to the basis n, p, taking into account their linear interdependence at fixed F, as was done in [21]. To save computer time, in most cases only two VRS iterations in item 3) are carried out, since its computational cost in solving the linear equations obtained from (12), (13) is considerable.

The two-stage VRS method considered can be generalized to any further shape of the recombination-generation law R (Q_n, Q_p, F), and to the case when Fermi statistics must be used. In this case the form of subsystems I and II (Eqs. (11)-(14)) changes: subsystem I acquires the form:

$$\left.\begin{aligned} W &= \tau_n [p(Q_p, F^{-1}) + 1] + \tau_p [n(Q_n, F) + 1], \\ W_p &= p(Q_p, F^{-1}) W^{-1},\ W_n = n(Q_n, F) W^{-1}, \end{aligned}\right\} \tag{15}$$

$$\nabla[\mu_p p(Q_p, F^{-1})\nabla \ln Q_p] - p(Q_p, F^{-1})W_n = -p_0 n_0 W^{-1} + R(Q_n, Q_p, F), \tag{16}$$

$$\nabla[\mu_n n(Q_n, F)\nabla \ln Q_n] - n(Q_n, F)W_p = -p_0 n_0 W^{-1} + R(Q_n, Q_p, F), \tag{17}$$

and subsystem II

$$\nabla^2(\ln F) = n(Q_n, F) - p(Q_p, F^{-1}) - N_d + N_a, \tag{18}$$

where the relation between n, p and Q_n, Q_p, F is given by the selected statistics. In that case the generalized two-staged VRS method will consist of the following:

1) the initial approximation for Ψ, Φ_n, Φ_p is given by the device structure;

2) subsystem II is solved by Newton's method for F at fixed Q_n and Q_p (as earlier);

3) subsystem I is solved by the VRS method for fixed F: a) W^l, W_p^l, W_n^l are found from (15); b) the quasilinear continuity equations (16), (17) are solved in $\Delta Q_p{}^l$, $\Delta Q_n{}^l$ for fixed $Q_n{}^l$, $Q_p{}^l$, W^l, $W_p{}^l$, $W_n{}^l$; c) new values of $Q_p{}^{l+1}$ and $Q_n{}^{l+1}$ are found; more precisely, $Q_p{}^{l+1} = Q_p{}^l + \Delta Q_p{}^l$, $Q_n{}^{l+1} = Q_n{}^l + \Delta Q_n{}^l$, $l = l + 1$, where l is the number of VRS iterations for subsystem II; d) in case of strong coupling between the continuity equations items a), b), and c) of 3) are repeated either a given number of times (usually twice) or till the required convergence (otherwise for $l = 1$ we leave the second stage);

4) items 2) and 3) of the two-stage VRS method are extended till the required accuracy.

The given method is a two-stage VRS method in the basis F, Q for any combination-generation law (Shockley—Read—Hall and others) and for the case of using Fermi statistics.

Consider the meaning of the suggested two-stage VRS methods. It is well known that physically the solution of the system of equations (1)-(5) must satisfy the conservation property, i.e., for specific Ψ

$$\nabla \vec{j} = \nabla(\vec{j}_n + \vec{j}_p) = 0. \tag{19}$$

However, during the iteration of the solution, as in the Seidman—Choo method [11], for example, Eq. (19) is written in the form:

$$(\nabla \vec{j})^n = \nabla(\vec{j}_n + \vec{j}_p)^n = R^n_{\text{Sh-R-H}} + S^n_n - R^n_{\text{Sh-R-H}} - S^n_p, \tag{20}$$

where $R^n_{\text{Sh-R-H}}$ is the numerical recombination value (6) (for the solution), and S^n_n, S^n_p are "numerical recombinations," related to errors in the initial approximations in all variables and the successive solution of the continuity equations (conservation in the approximate problems is guaranteed by using the Marchuk integral identity [4]). Thus, generally speaking, during the solution iteration the conservation property (19) breaks down, since $S^n_n - S^n_p \neq 0$. This is unavoidable for all single-stage methods in which the continuity equations are solved successively.

Consider now the first two-stage VRS method.

During its application in item 3) for given F, as a result of the convergence of iterations Q_p, Q_n, W, WQ_p, WQ_n tend to constant values, and, consequently, $[\nabla \vec{j}_n(F)]^n = -[\nabla \vec{j}_p(F)]^n$, since $Q_n W Q_p = Q_p W Q_n$ ($R^n_n = R^n_p$). Thus, the basic meaning of the second stage of iteration of the VRS method is the satisfaction of the conservation property at each step of the solution (iteration).

This result can be summarized in form of a principle of physical balance of the iteration solution of the problem: to accelerate the convergence of iteration methods of solution of problems describing transfer processes in semiconducting devices it is necessary, as much as possible, to supplement balancing of the iteration solution of the problem while observing the conservation property. Thus, the solution of the problem is sought within a given quality (conservation). A quantitative measure for deviation from this principle can be provided by "numerical sources" in the form of "numerical recombination," related to successive solutions of the continuity equations and to errors in the initial approximations for Q_n, Q_p, and F.

To obtain "exact" solutions it is necessary that F tend to the "exact" value. The use of the concept of "numerical sources" in the form of "numerical recombination" and "unbalanced charge" is also useful for the given problem, as was shown in [21], for a qualitative theoretical study of the convergence of iteration methods.

Thus, a general method was suggested for constructing effective and reliable numerical methods and algorithms for analyzing semiconducting devices and structures. Its effectiveness

will be illustrated in a future publication by a numerical experiment on the example of analyzing the operation of a single-collector cell of integrated engineering logic (IEL).

NOTATION

Ψ, electrostatic potential; n and p, electron and hole concentrations, respectively; N_d and N_a, donor and acceptor concentrations; $\vec{j}_n$, $\vec{j}_p$, and $\vec{j}$, vector densities of the electron current, hole current, and total current; R, excess rate of recombination over generation for holes (R_p) and electrons (R_n); q, electron charge; ε, dielectric constant of the material; μ_n and μ_p, electron and hole mobilities; Φ_n and Φ_p, electron and hole Fermi quasilevels; n_0 and p_0, electron and hole concentrations at thermodynamic equilibrium; n_i, intrinsic concentration; τ_n and τ_p, electron and hole lifetimes; kT/q, temperature potential (equal to 0.0258 V at $T = 300^\circ K$); x_k and x_{k+1}, sites of the basic grid of spatial discretization; $x_{k-1/2}$ and $x_{k+1/2}$, sites of auxiliary grid of spatial discretization; h_k and h, grid steps for spatial discretization ($h_k = x_{k+1/2} - x_{k-1/2}$); y_{k+1} and $y_{k+1/2}$, values of the function y at the points x_{k+1} and $x_{k+1/2}$; $\delta\Psi$, perturbation to Ψ, relatively to which is formulated the linearized Poisson equation ($\delta\Psi$ and other unknowns are, rigorously speaking, vectors, when the topic is vector relaxation); $(A)^n$, numerical values of the corresponding variables at the grid sites of spatial discretization; and p^M, q^M, φ^M, and f^M, functions in the G. I. Marchuk approximate problem (Eq. (1.12), [14]).

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A SYSTEM FOR COMPUTER-AIDED THERMAL DESIGN

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UDC 536.24:681.2

A systems approach and design unification are used in a method of automating the thermal design of electronic devices.

The design of a device may involve electrical, mechanical, optical, and other systems, and many specification have to be met: functional, technological, working, reliability, economic, etc. These requirements can be optimally met by means of two major principles used in computer-aided design (CAD) systems: design standardization and a systems approach. The devices may be based on a modular principle, which involves a hierarchic tree, i.e., a device is divided into several units, which themselves are divided into units of lower rank. Therefore, the structure is to be considered as components that are largely standardized and have a hierarchic relation (electronic components, cassettes, boards, blocks, cooling system, etc.).

In a systems approach to design, the individual sections or devices are considered as a whole; characteristic units are identified together with the links between them, as well as the effects of changes in individual components on device operation; optimum design is applied to the architecture, followed by stepwise optimization of the various units.

It must be emphasized that any deviation from integrity in the approach, with a desire to work out certain design problems to completion, while the others are left aside and considered only if necessary, will result in violation of the information links between the subsystems, which subsequently will require much time in correcting and finishing the device.

Design automation involves a set of linked problems [1], one of which is the computer-aided thermal design (CATD) system, which is a subsystem in the general computer-aided design (CAD) system. The CATD includes ways of simulating the temperature distributions in complicated devices, methods of automating the thermal calculation for various working conditions, and computer organization of the thermal-design system.

The thermal design is closely related to other subsystems and is realized at various stages: in simulating the functions, in defining the components, in locating them, and in providing the technology.

At the simulation stage, one analyzes the circuit and identifies the overloaded points by considering the heat produced for each component.

In selecting the components [2] from a given logical description of the system as a whole, a hierarchic structure must be set up such that any subsystem in the first level is contained in a rack, section, panel, etc. of given size. Then the units of the second rank (cassettes and blocks) and those in the third rank (boards containing micromodules, integrated circuits,

Leningrad Fine Mechanics and Optics Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 44, No. 2, pp. 293-298, February, 1983. Original article submitted October 28, 1981.

 0022-0841/83/4402-0206$07.50